Recon Core subproject III

MRI Reconstruction Using Graph Cuts:

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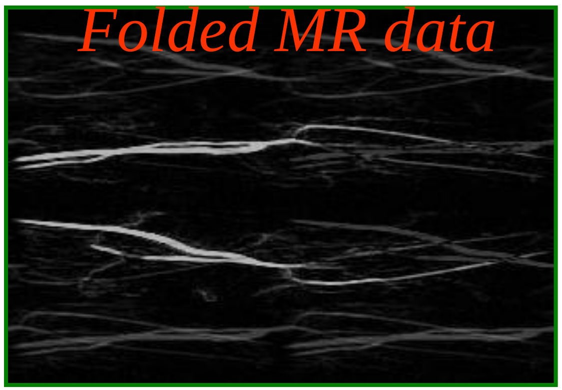
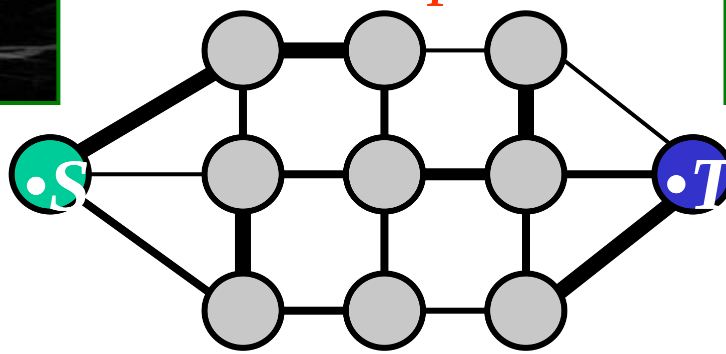
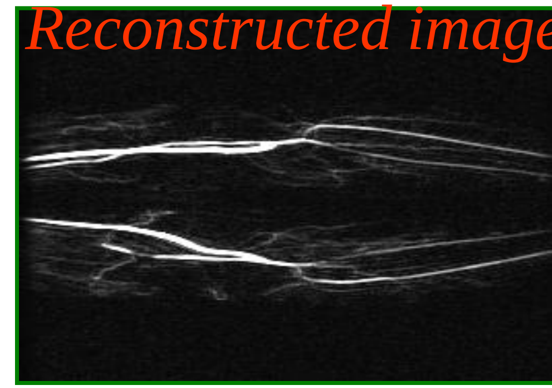
- *A new graph-based algorithm* *
- *Inspired by advanced robotic vision, computer science*



•Graph





- *Operations on this graph produce reconstructed image!*

•Raj et al, Magnetic Resonance in Medicine, Jan 2007,

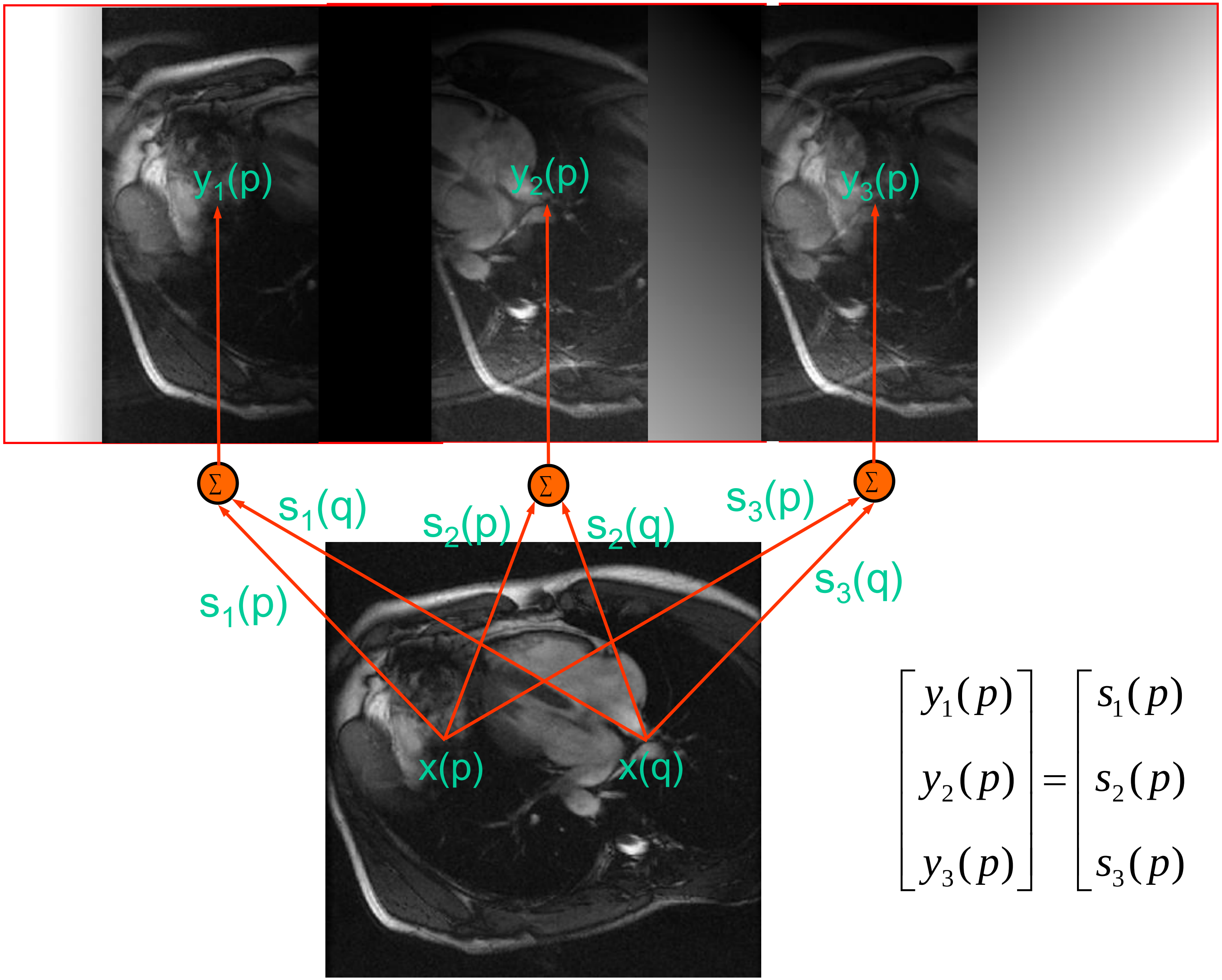
•Raj et al, Computer Vision and Pattern Recognition, 2006

•Singh et al., MRM (to appear)

Project Summary

- Aim1: To apply EPIGRAM to fast high-resolution structural brain imaging
  - Image priors to be empirically evaluated
- Aim 2: Extending the method from 2D to 2D + time data
- Aim 3: Validation
- Aim 4: Developing new efficient, feasible Graph algorithms

Significant advances were made in all aims (except Aim 3)



$$\begin{bmatrix} y_1(p) \\ y_2(p) \\ y_3(p) \end{bmatrix} = \begin{bmatrix} s_1(p) & s_1(q) \\ s_2(p) & s_2(q) \\ s_3(p) & s_3(q) \end{bmatrix} \begin{bmatrix} x(p) \\ x(q) \end{bmatrix}$$

Least squares solution

- Least squares estimate:

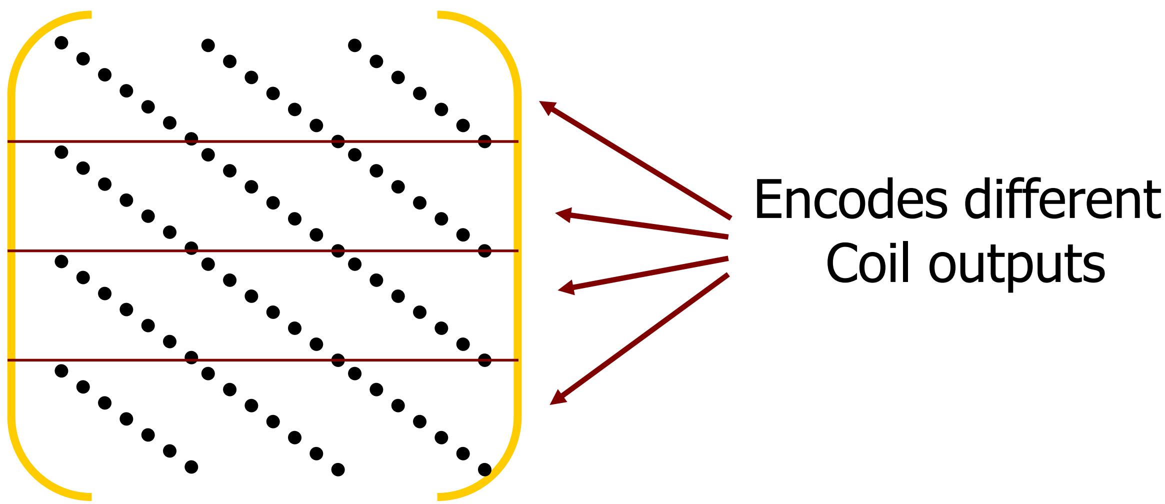
$$(\hat{x}(p), \hat{x}(q)) = \arg \min_{x(p), x(q)} \sum_{l \in \text{Coils}} \left[y_l(p) - s_l(p)x(p) - s_l(q)x(q) \right]^2$$

  – Famous MR algorithm: SENSE (1999)

- Linear system

$$y = Hx + n$$

$$H =$$



EPIGRAM Summary

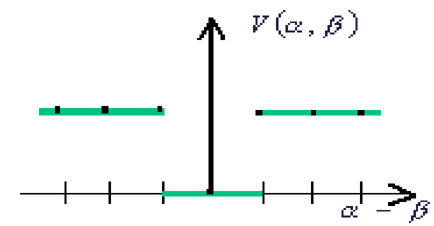
- Finds the MAP estimate

$$\hat{x} = \arg\min_{x} E(x) \equiv \left[\|y - Hx\|^2 + \lambda G(x) \right]$$

Makes Hx close to y

Makes x piecewise smooth

Potts function

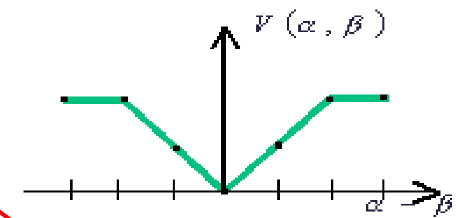


- Used Markov Random Field priors

$$G(x) = \sum_{(p,q) \in \mathcal{N}} V(x_p - x_q)$$

–If V "levels off", this preserves edges

Truncated L1 distance



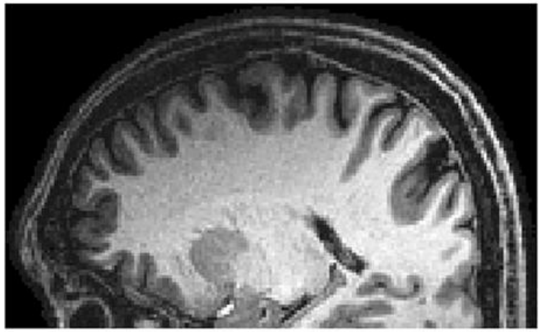
Robust

New Developments (I):

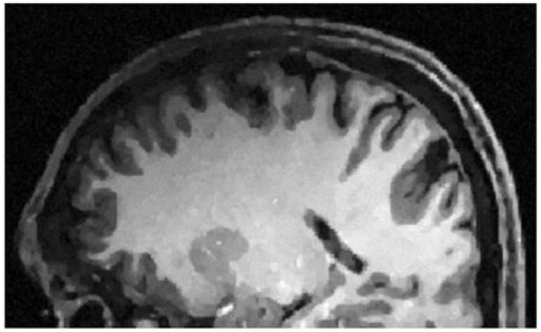
- Extension of EPIGRAM from 2D to 3D
- Phase-constrained reconstruction

Phase Constrained EPIGRAM: Reff = 4.5

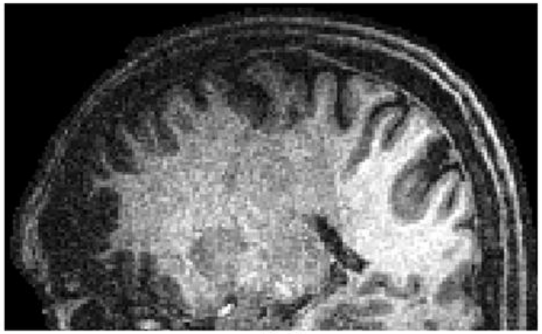
Reference:
Sum of squares



(a)



(b)



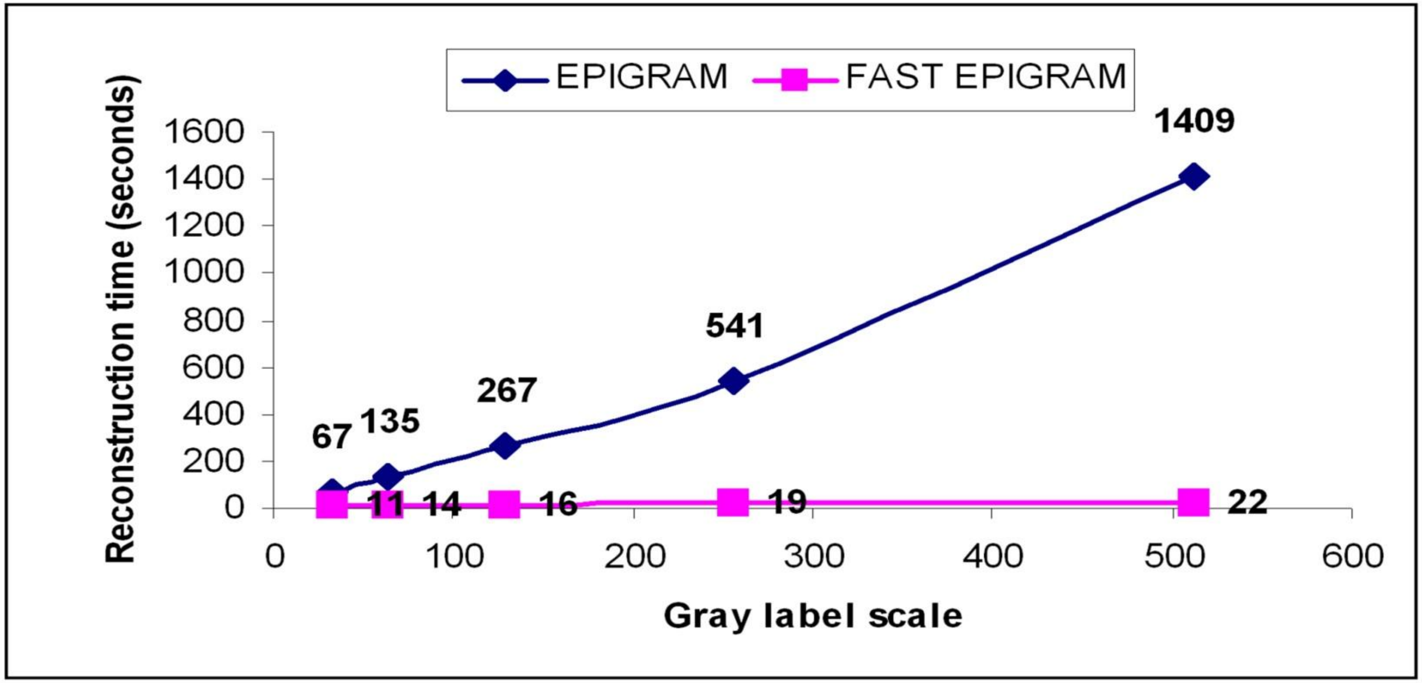
(c)

Fast EPIGRAM Regularized SENSE

New Developments (II):

- Fast EPIGRAM – uses “jump moves” rather than “expansion moves”
  - Up to 50 times faster!

New, Faster Graph Cut Algorithm: Jump Moves



•*Reconstruction time of EPIGRAM (alpha expansion) vs Fast EPIGRAM (jump move)*
- *after 5 iterations over [32, 64, 128, 256, 512] gray scale labels*
- *image size 108x108 pixels.*

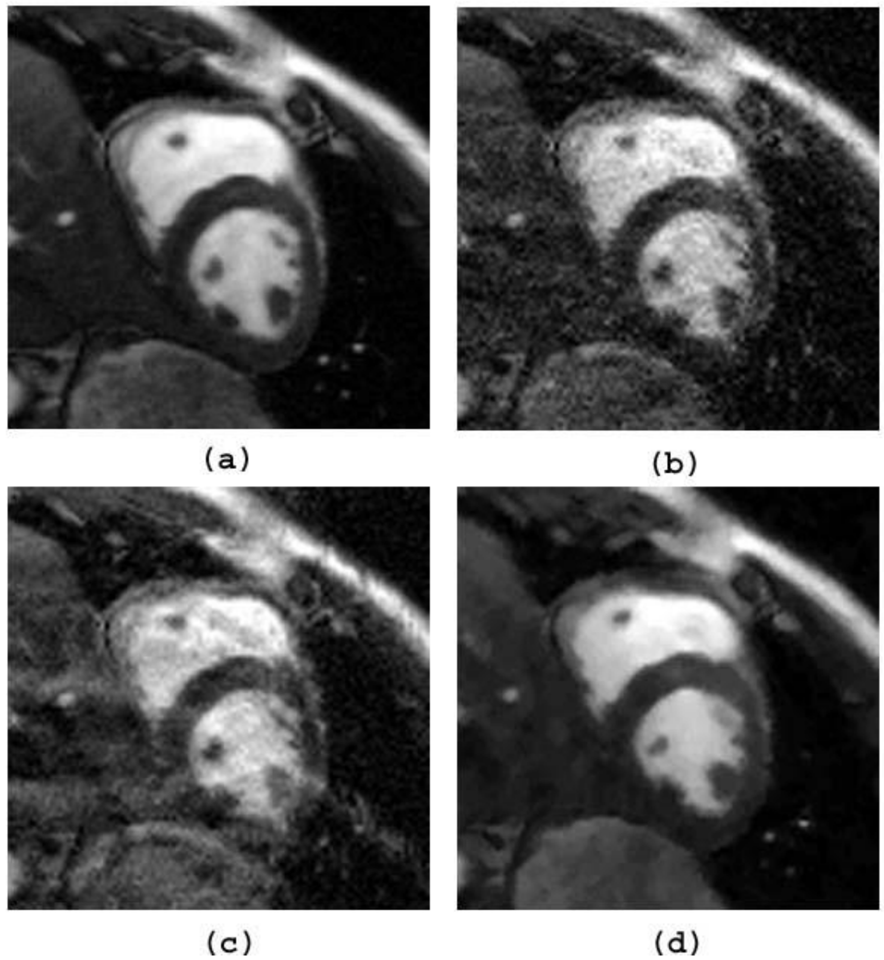
• *Linear versus exponential growth in in reconstruction time*

Jump Move Results: Cardiac Imaging, R=4

• *reconstruction for cine SSFP at R = 4*

Reference:
Sum of squares

Regularized SENSE
($\mu = 0.1$)



Regularized SENSE
($\mu = 0.5$)

Fast EPIGRAM

New Developments (III):

Automatically Learning Image Priors

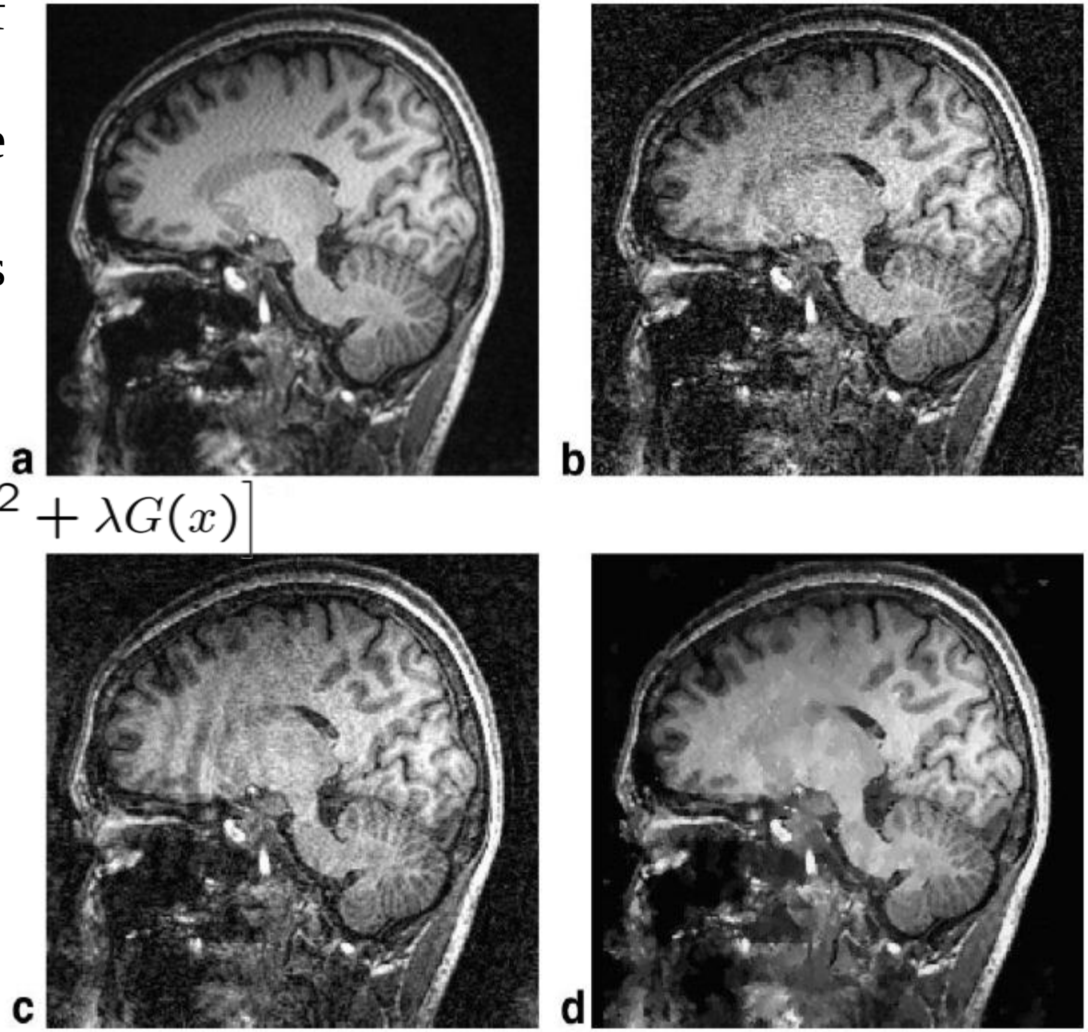
- The most important aspect of EPIGRAM is choice of prior
- What is the most appropriate prior model?
- Recon performance depends crucially on prior model
- Recall:

$$\widehat{x} = \arg\min_x E(x) \equiv \left[\|y - Hx\|^2 + \lambda G(x)\right]$$

$$G(x) = \sum_{(p,q)\in\mathcal{N}} V(x_p - x_q)$$

Form of V determines recon image

$V(\delta) = 0.1|\delta|^2$



$V(\delta) = 0.2|\delta|^2$

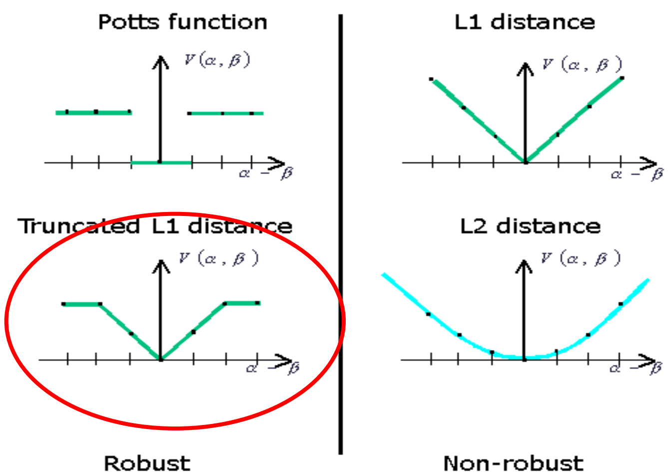
$V(\delta) = 0.1\min(50, |\delta|)$

New Developments (III):

Automatically Learning Image Priors

- New idea: automatically learning what prior model best fits brain MRI
- Generalization of edge-preserving Gibbs priors successful in EPIGRAM
- Define a class of prior distributions ➔ mixture of various powers

$$G(x) = \sum_{(p,q)\in\mathcal{N}} V(x_p - x_q)$$



Original prior model

image

Diff image
$\delta = x_1 - x_2$

$$Pr(x) = Pr(\delta) \propto \exp\left(-\frac{1}{2M}\sum_{\delta} V_{\gamma,\alpha,K}(\delta)\right)$$

$$V_{\gamma,\alpha,K} = \sum_{i\in\gamma} \frac{\alpha_i \min(\delta^{p_i}, K_i^{p_i})}{p_i}.$$

exponent

cutoff

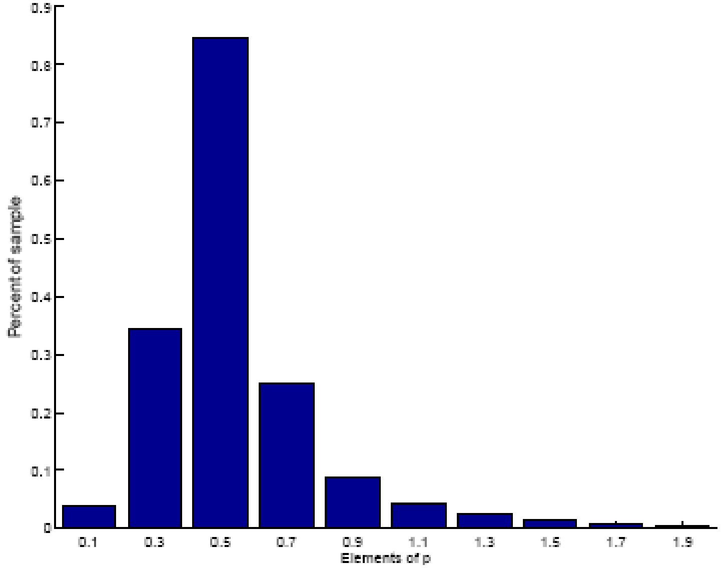
Mixture weight

Proposed prior model

Learning Image Priors: Technique

- Used Markov chain Monte Carlo (McMC) technique to learn unknown parameters of prior model
- McMC sampling is based on Metropolis-Hastings algorithm
- After 1000s of iterations, gives a posterior distribution of the model
- We use the maximum of this inferred posterior

Histogram of various exponents "visited" by McMC sampler



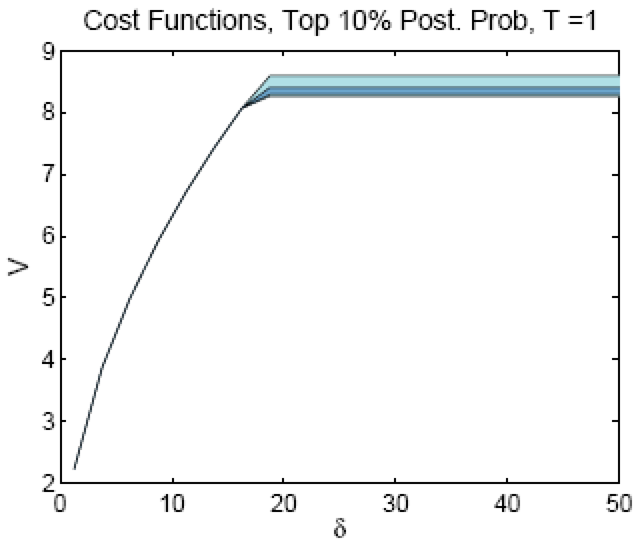
Learning Image Priors: Results

- Found a strong maximum of posterior
- Inferred model: exponent = 0.5, cutoff = 25

exponent

$$V_{\gamma,\alpha,K} = \min(\delta, 25.42)^{0.5}$$

cutoff



We believe this prior will be superior to previous prior
Results on brain data awaited

Learning Image Priors: Simulations

- Shepp-Logan head phantom with different noise and blur (PSF)
- Width of Gaussian blur kernel: 0, 2, 4, 6, 8, 10, 20
- Inferred model should depend on size of blur
    - (more blurry image ➔ higher exponent)

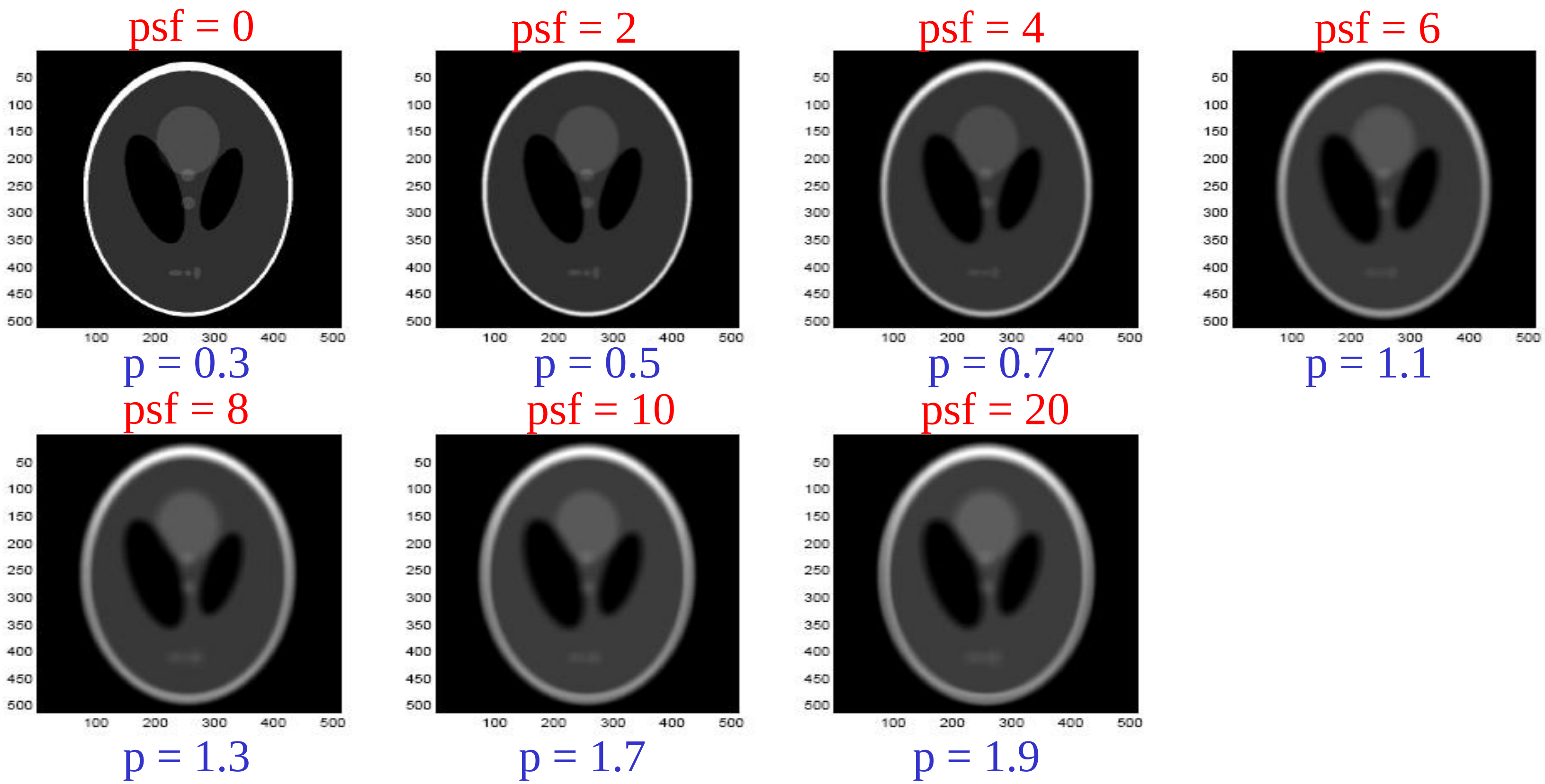


Figure 1: The different images corresponding to the different widths of Gaussian blur.

Learning Image Priors: Simulations

Result: found almost linear dependency

Blur Width	Parallel Tempering, T = 1	Regular Sampling, T = 1
0	$\min(\delta, 57.70)^{0.3}$	$\min(\delta, 55.67)^{0.3}$
2	$\min(\delta, 17.57)^{0.5}$	$\min(\delta, 16.56)^{0.5}$
4	$\min(\delta, 7.79)^{0.7}$	$\min(\delta, 7.77)^{0.7}$
6	$\min(\delta, 4.46)^{1.1}$	$\min(\delta, 4.47)^{0.9}$
8	$\min(\delta, 3.14)^{1.3}$	$\min(\delta, 3.07)^{1.3}$
10	$\min(\delta, 2.24)^{1.7}$	$\min(\delta, 2.21)^{1.7}$
20	$\min(\delta, 1.93)^{1.9}$	$\min(\delta, 1.91)^{1.9}$
Brain MRI	$\min(\delta, 11)^{0.5}$	$\min(\delta, 11.67)^{0.5}$

Table 1: The estimate of the MAP for the 7 different blurred Shepp-Logan Phantoms, as well as brain MR images.

New Developments (IV):

- New graph cut algorithm to replace EPIGRAM – does not use expansion moves at all
- Expect o(10-100x) computational speed up
- Based on exploring null-space of system matrix H

$$E(x) \equiv \left[\|y - Hx\|^2 + \lambda G(x)\right]$$

- Let D = *null(H)*, x_0 be any solution to $y = Hx$
- Let $x = x_0 + D\eta$. Then $y - Hx = y - H x_0$ and $E(x) = \lambda G(\eta)$
- Henceforth we seek graph cut moves on η rather than x
- Since nullspace is much smaller than space of x, this is much more efficient